

STEPENOVANJE

Proizvod $a \cdot a \cdot \dots \cdot a = a^n$ naziva se n -tim stepenom broja. Ako je $a \in R$, $a \neq 0$ i neka je $n \in N$
 n -puta

Po definiciji je:

$$1) a^0 = 1 \rightarrow \text{primer: } 5^0 = 1, (-3)^0 = 1, \left(\frac{4}{7}\right)^0 = 1$$

$$2) a^{-n} = \frac{1}{a^n} \rightarrow \text{primer: } 3^{-2} = \frac{1}{3^2} = \frac{1}{9}, 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

Još važe sledeća pravila:

$$3) a^m \cdot a^n = a^{m+n} \rightarrow \text{primer: } 3^2 \cdot 3^5 = 3^{2+5} = 3^7$$

$$4) a^m : a^n = a^{m-n} \rightarrow \text{primer: } 7^{10} : 7^6 = 7^{10-6} = 7^4$$

$$5) (a^m)^n = a^{m \cdot n} \rightarrow \text{primer: } (2^3)^5 = 2^{3 \cdot 5} = 2^{15}$$

$$6) (a \cdot b)^n = a^n \cdot b^n \rightarrow \text{primer: } (12 \cdot 11)^5 = 12^5 \cdot 11^5$$

$$7) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \rightarrow \text{primer } \left(\frac{7}{4}\right)^2 = \frac{7^2}{4^2}$$

$$8) \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \rightarrow \text{primer } \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$$

O čemu treba voditi računa?

Treba paziti na zapis: $(-5)^2 = (-5)(-5) = 25$, dok $-5^2 = -5 \cdot 5 = -25$. Uopšteno važi:

$$(-a)^{\text{paran}} = a^{\text{paran}}$$

$$(-a)^{\text{neparan}} = -a^{\text{neparan}}$$

Dakle, paran izložilac ‘uništi’ minus.

ZADACI

1) Izračunati: $\frac{(2^7 : 2^5) \cdot 2^3}{2^4 : 2^2}$

$$\frac{(2^7 : 2^5) \cdot 2^3}{2^4 : 2^2} = \frac{2^{7-5} \cdot 2^3}{2^{4-2}} = \frac{2^2 \cdot 2^3}{2^2} = \frac{2^{2+3}}{2^2} = \frac{2^5}{2^2} = 2^{5-2} = 2^3 = 8$$

2) Izračunati: $\frac{3^5 \cdot 9^3}{27^2 \cdot 3}$

$$\frac{3^5 \cdot 9^3}{27^2 \cdot 3} = \frac{3^5 \cdot (3^2)^3}{(3^3)^2 \cdot 3^1} = \frac{3^5 \cdot \cancel{3^6}}{\cancel{3^6} \cdot 3^1} = \frac{3^5}{3^1} = 3^{5-1} = 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

3) Izračunati: $\frac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3} =$

$$\frac{(x^4)^3 \cdot x^3 : x^5}{(x^5 : x^2)^3} = \frac{x^{12} \cdot x^3 : x^5}{(x^{5-2})^3} = \frac{x^{12+3-5}}{(x^3)^3} = \frac{x^{10}}{x^9} = x^{10-9} = x^1 = x$$

4) Izračunati: $\frac{3^{n+1} \cdot 3^{n+2}}{3^{2n+4}}$

$$\frac{3^{n+1} \cdot 3^{n+2}}{3^{2n+4}} = \frac{3^{n+1+n+2}}{3^{2n+4}} = \frac{3^{2n+3}}{3^{2n+4}} = \text{Pazi pa zagrade zbog minusa}$$
$$= 3^{(2n+3)-(2n+4)} = 3^{2n+3-2n-4} = 3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

5) **Izračunati** $0,5^{-1} + 0,25^{-2} + 0,125^{-3} + 0,0625^{-4}$

$$0,5^{-1} + 0,25^{-2} + 0,125^{-3} + 0,0625^{-4} =$$

$$\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{8}\right)^{-3} + \left(\frac{1}{16}\right)^{-4} =$$

$$\left(\frac{2}{1}\right)^1 + \left(\frac{4}{2}\right)^2 + \left(\frac{8}{1}\right)^3 + \left(\frac{16}{1}\right)^4 =$$

$$2^1 + 4^2 + 8^3 + 16^4 = 2 + 16 + 512 + 65536 = 66066$$

6) **Izračunati** $1^{-1} + 2^{-2} + 3^{-3} + (-1)^{-1} + (-2)^{-2} + (-3)^{-3}$

$$1^{-1} + 2^{-2} + 3^{-3} + (-1)^{-1} + (-2)^{-2} + (-3)^{-3} =$$

$$\frac{1}{1} + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{(-1)^1} + \frac{1}{(-2)^2} + \frac{1}{(-3)^3} =$$

$$1 + \frac{1}{4} + \frac{1}{27} - 1 + \frac{1}{4} - \frac{1}{27} = \frac{1}{4} + \frac{1}{4} - \frac{1}{27} + \frac{1}{27} = \frac{2}{4} = \frac{1}{2}$$

7) Ako je $a = 5^3 \cdot \left(\frac{1}{4}\right)^{-4} \cdot \left(\frac{3}{2}\right)^2$ i $b = 10^3 \left(\frac{5}{3}\right)^{-2}$ najdi $a \cdot b^{-1}$

$$a = 5^3 \cdot \left(\frac{1}{4}\right)^{-4} \cdot \left(\frac{3}{2}\right)^2 = 5^3 \cdot \left(\frac{4}{1}\right)^4 \cdot \frac{3^2}{2^2} = \frac{5^3 \cdot 4^4 \cdot 3^2}{2^2}$$

$$= \frac{5^3 \cdot (2^2)^4 \cdot 3^2}{2^2} = 5^3 \cdot (2^2)^3 \cdot 3^2 = 5^3 \cdot 2^6 \cdot 3^2$$

$$b = 10^3 \cdot \left(\frac{5}{3}\right)^{-2} = 10^3 \cdot \left(\frac{3}{5}\right)^2 = \frac{10^3 \cdot 3^2}{5^2} = \frac{(5 \cdot 2)^3 \cdot 3^2}{5^2} = \frac{5^3 \cdot 2^3 \cdot 3^2}{5^2} = 5 \cdot 2^3 \cdot 3^2$$

Konačno: izračunati i $a \cdot b^{-1}$

$$a \cdot b^{-1} = 5^3 \cdot 2^6 \cdot 3^2 \cdot \frac{1}{5 \cdot 2^3 \cdot 3^2} = 5^2 \cdot 2^3 = 25 \cdot 8 = 200$$

8) Izračunati $\left(\left(\frac{5x^{-5}}{2y^{-2}} \right)^{-2} \cdot \left(\frac{y^{-1}}{5x^{-1}} \right)^{-3} \right) : 10x^2y^{-3}$

$$\left(\left(\frac{5x^{-5}}{2y^{-2}} \right)^{-2} \cdot \left(\frac{y^{-1}}{5x^{-1}} \right)^{-3} \right) : 10x^2y^{-3} =$$

$$\left(\frac{5^{-2} \cdot x^{10}}{2^{-2} \cdot y^4} \cdot \frac{y^3}{5^{-3} \cdot x^3} \right) : 10x^2y^{-3} =$$

$$(5^{-2+3} \cdot x^{10-3} \cdot y^{3-4} \cdot 2^2) : 10x^2y^{-3} =$$

$$(5^1 \cdot x^7 \cdot y^{-1} \cdot 4) : 10x^2y^{-3} =$$

$$\frac{20}{10} x^{7-2} y^{-1-(-3)} = 2x^5 y^{-1+3} = 2x^5 y^2$$

9) Ako je $10^x = \frac{\frac{1}{2}10^{-3} + \frac{1}{2}10^{-4}}{55 \cdot 10^{-7}}$ **Odrediti x.**

$$\frac{\frac{1}{2}10^{-3} + \frac{1}{2}10^{-4}}{55 \cdot 10^{-7}} = \frac{\frac{1}{2000} + \frac{1}{20000}}{55} = \text{Izvučemo gore zajednički}$$

$$\frac{\frac{1}{2000} \left(1 + \frac{1}{10} \right)}{55} = \frac{\frac{1}{2000} \cdot \frac{11}{10}}{55} = \frac{\frac{11}{20000}}{55} =$$

$$\frac{\frac{11 \cdot 10000000}{20000 \cdot 55}}{11 \cdot 100000} = \frac{11 \cdot 10000000}{11 \cdot 100000} = \frac{10000000}{100000} = 100 = 10^2$$

sada je $10^x = 10^2$, dakle $x = 2$

10)

a) $A \cdot 10^{-5} = 0,2 \cdot 0,008$

$$A \cdot 10^{-5} = 2 \cdot 10^{-1} \cdot 8 \cdot 10^{-3}$$

$$A \cdot 10^{-5} = 16 \cdot 10^{-4}$$

$$A = \frac{16 \cdot 10^{-4}}{10^{-5}}$$

$$A = 16 \cdot 10^{-4-(-5)}$$

$$A = 16 \cdot 10^{-4+5}$$

$$A = 16 \cdot 10$$

$$A = 160$$

b) $B \cdot 10^{-6} = 0,04 \cdot 0,006$

$$B \cdot 10^{-6} = 4 \cdot 10^{-2} \cdot 6 \cdot 10^{-3}$$

$$B \cdot 10^{-6} = 24 \cdot 10^{-5}$$

$$B = \frac{24 \cdot 10^{-5}}{10^{-6}}$$

$$B = 24 \cdot 10^{-5+6}$$

$$B = 24 \cdot 10$$

$$B = 240$$